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Kanellopoulos, Vassilis (GR-ATHN)

Ramsey families of subtrees of the dyadic tree. (English summary)

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The main result of this article, Theorem A, concerns rooted trees which have height ω, are finitely branching, and have no maximal elements: For every such tree T there is a family $\mathcal{F}$ of (set-theoretical) subtrees of the dyadic tree C (C has height ω and every vertex in C has two children) which are order-isomorphic to T and are such that (i) $\mathcal{F}$ is a G_δ subset of 2^C; (ii) every perfect subtree D of C (i.e., every $v \in D$ has at least two incomparable successors) satisfies $D \supset U$ for some $U \in \mathcal{F}$; (iii) if $K \subset \mathcal{F}$ is an analytic subset and $S \subset C$ is a perfect subtree then there is a perfect subtree $S' \subset S$ such that the set $\{A \in \mathcal{F} : A \subset S'\}$ is either contained in K or is disjoint with K.

Kanellopoulos proves Theorem A by means of the abstract Ramsey theory due to S. Todorčević (he briefly recalls it in Section 4) and by means of the following finite-type Ramsey result for infinite trees, Theorem B, which he derives from the Halpern-Läuchli theorem. Take $d+1$ dyadic trees $T, S_1, \dots, S_d$ and finitely-color the set X of all $(d+1)$-tuples $(t, u_1, \dots, u_d)$ where $t \subset T(n)$, $|t| = 2$, $u_i \in S_i(n)$, and $n \geq 0$ ($T(n)$ and $S_i(n)$ are the n-th levels of the dyadic trees). Then there exists a sequence of integers $0 \leq m_0 < m_1 < \dots$ and dyadic subtrees $T' \subset T$, $S'_i \subset S_i$ such that $T'(n) \subset T(m_n)$, $S'_i(n) \subset S_i(m_n)$ for $n \geq 0$ and the set of $(d+1)$-tuples $X' \subset X$ corresponding to $T', S'_1, \dots, S'_d$ is monochromatic.

Martin Klazar

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